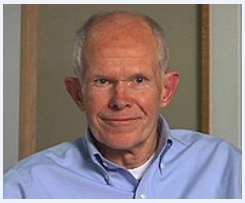
DOINGWHATWORKS

VIDEO
6:51 MIN

Full Details and Transcript



Building on Intuitive Understanding

Thomas P. Carpenter, Ph.D.
January 2011

Topic DEVELOPING EFFECTIVE FRACTIONS INSTRUCTION FOR K-8

Practice INITIAL FRACTION CONCEPTS

Highlights

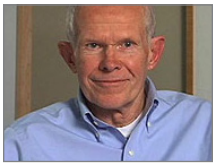
- Dr. Carpenter discusses the importance of helping children understand the big ideas about fractions.
- Beginning with the concept of fair sharing, he demonstrates how children might address a problem that involves distributing sets fairly among children, including how they might draw a representation and count the number in each set.
- He demonstrates the next progression of difficulty in fair sharing, which is distributing an uneven number of objects among children.
- We see that one child finds the solution of halving or portioning an object.
- Dr. Carpenter demonstrates ways to help children see equivalent shares that are represented in different ways.
- In a problem more suited for older children, he demonstrates how to successively partition objects among a number of children using different seating arrangements to share varying amounts of food.
- The conclusion is a plea for encouraging educators to allow children to use their informal understandings of sharing to explore different

problems and explain their solutions using manipulatives and representations.

About the Interviewee

Thomas Carpenter is Emeritus Professor of Curriculum and Instruction (Mathematics Education) at the University of Wisconsin–Madison and director of diversity in mathematics education at the Center for Learning and Teaching. He served as director of the National Center for Improving Student Learning and Achievement in Mathematics and Science and as editor of the *Journal for Research in Mathematics Education*. His research integrates the study of the development of children's mathematical thinking, instruction that supports that development, and professional development that fosters instruction that leads to learning with understanding. His recent research focuses on the development of algebraic thinking in elementary school, in particular the development of relational thinking, generalization, mathematical representations, and proof.

Full Transcript



00:00 I'm Tom Carpenter. I'm an emeritus professor at the University of Wisconsin–Madison and a researcher at the Wisconsin Center for Education Research. I was a member of the What Works Clearinghouse panel that produced the report *Developing Effective Fraction Instruction*.

00:22 I'm going to talk about the first recommendation in the report, which is to build upon children's intuitive knowledge of sharing to develop fraction concepts. The overarching theme of the report is that effective instruction is grounded in developing understanding. The defining feature of learning with understanding is that knowledge is connected. In particular, it's important that knowledge is connected to the things that children already understand.

00:47 Young children have a lot of intuitive knowledge about sharing situations and can solve problems involving sharing. And these sharing problems can be used then to develop basic fraction concepts. We usually start, essentially, with division. Children as young as

kindergarten can solve division problems involving sharing that will ultimately lead up to developing fraction ideas.

01:13 For example, you might pose the following problem to young children: "I just baked 12 cupcakes, and I want to put them on three plates. How many cupcakes would be on each plate?" So children might solve that problem by starting out and counting out counters of some kind to represent the 12 cupcakes. Children might draw some representation of the plates or they might use some other way of representing the plates, but very commonly they'll draw plates like that. And then they will essentially say, "Well, we've got to have the same number on each plate, so I will deal the counters to the plate, one at a time." And then they would count the number of cupcakes on each plate—one, two, three, four; one, two, three, four; one, two, three, four—probably count them all. They could just count one plate, since they're all the same, but they'll count them all.

02:07 So they start with problems like that, and that's a basic division problem that children in kindergarten can solve. They don't solve that because they have been shown how to solve them; these are sort of intuitive solutions. And it's really much more effective to pose the problems to the children and let them come up with their specific ways of solving them.

02:28 After children have had some experience partitioning sets that can be partitioned exactly, then we pose a problem to them like "we have two children sharing three cupcakes." Now you can't give all the cupcake; you can't take the three cupcakes and put them into even piles. Children have to figure out what to do with the leftover cupcake. A solution that the child herself drew—this is a first-grade child—drew the two people sharing the cupcakes, drew the three cupcakes, and then gave one cupcake to each child, and then divided the remaining cupcake in half.

03:12 The notion of halving can be used to solve quite a few problems, like we've just seen, but it's somewhat limited in terms of developing fraction concepts. So here's another example, where three

children were asked to share five cakes, and each child was given one cake. The other two cakes were cut in half, so each child could get another half. But this particular child—a first grader—didn't know what to do with the remaining part. The remaining part, he would give that to the teacher or give that to someone else. One of the things children need to get is some sort of an anticipatory notion—to anticipate how many pieces they're going to cut it into and sort of start out thinking about the number of pieces that they want to get.

04:03 One of the central ideas in understanding fractions is the notion of equivalence. Equivalence underlies most of the operations that you do with fractions. If you really understand equivalence, then adding and subtracting fractions becomes fairly straightforward.

04:19 In this example, we have eight children sharing five pancakes. And one child did it by cutting each of the pancakes up into eight pieces, and so each child got five-eighths. In the other example, another child cut each of the pancakes into halves, and each child got a half. But then there was one pancake that needed to be cut up some more, and so the child cut it up into halves of the halves and halves of the halves again, so there were eight pieces, and wound up with the solution of one-half plus one-eighth. And so there the issue then became who gets more, the child who cut five-eighths or the child who got one-half plus one-eighth? What's the relationship between one-half and the four-eighths that are compared? So again, you're comparing equivalent fractions.

Older children

05:13 Sharing can also be used with older children to further develop the idea of equivalence. Older children won't necessarily draw pictures to represent the problems; they can use numerical representation of the problems. Here is a representation that was developed by a Dutch researcher by the name of Leen Streefland for a problem he posed of 24 children going to a pizza parlor and ordering 16 pizzas. So the question is, "How can these children sit at different table arrangements so that they all get the same amount of pizza?"

So one possibility is that 24 children sit at one table and share the 16 pizzas. But another possibility is that we divide the table down so that we have two tables of 12 children with eight pizzas on each table. Or we can divide those tables down further so that six children are sharing four pizzas, and ultimately that three children are sharing two pizzas. Each of these situations are essentially representations of equivalent fractions and linked into the ideas of sharing.

06:21 What we see in these examples are children solving problems in ways that make sense to them. They have some intuitive knowledge of sharing. And the teacher has posed fraction problems in terms of sharing situations that the children are allowed to solve in ways that they devise for themselves. You pose the problems and get out of the way and let the children solve them and to build on the knowledge that they bring to instruction.